Adaptive Observers and Estimation of the Road Profile

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ABSTRACT

In this paper, we present an adaptive observer to estimate the unknown parameters of a vehicle. The system unknown inputs, representing the road profile variations, are estimated using sliding mode observers. First, we present some results related to the validation of a full car modelization, by means of comparisons between simulations results and experimental measurements (coming from a Peugeot 406 as a test car). Because, we don't know exactly pneumatic parameters and because these parameters can be changed, an other sliding mode observer is developed to estimate the longitudinal forces (which depend on these parameters) acting on the wheels. The estimated Road Profile is compared to the measured one coming from the LPA (longitudinal profile analyser ) in order to test the robustness of our approach.

INTRODUCTION

The knowledge of road profile represents basic information for studying vehicle-road interactions.

In order to have this road profile, several methods have been developed. Direct measurements of the road roughness using profiling instruments are proposed by different laboratories. The Roads and Bridges Central Laboratory (in French : LCPC) has developed a longitudinal profile analyser (LPA) ([1]). It is equipped with laser sensor to measure the elevation of the road profile. A profiler is an instrument used to produce a series of numbers related in a well-defined way to a true profile ([2]). However, this instrument does not measure true profile exactly. Other geometrical methods using many sensors (distance sensors, accelerometers ..) were developed ([3][4]). However, these methods depend directly on the sensors reliability and cost. It is worthwhile to mention that these methods do not take into consideration the dynamical behaviour of the vehicle. However, it has been shown ([5]) that modifications of the dynamical behaviour may lead to biased results.

In this paper, we present a method to estimate the road profile by means of sliding mode observers designed from a dynamic modelization of the vehicle ([6]). We estimate the unknown inputs of the system corresponding to the elevation of the road. Some pneumatic parameters are not known, that is why we develop a second observer to estimate the longitudinal forces which depend on these parameters ([7]). First, we introduce a full car modelization with 16 degrees of freedom (see [8]). The model validation is done through comparisons between simulation results and experimental ones coming from a test car ([9]).

This paper is organized as follows: section 2 deals with the vehicle description and modelling then some comparison results are presented to evaluate the accuracy of the model. The design of the sliding mode observer is presented in section 3. Some results about the states observation and the road profile estimation by means of the proposed method are presented in section 4. At last but not least, a conclusion is drawn in section 5.

VEHICLE DYNAMIC MODEL

When considering the vertical displacement along Z axis, the dynamic model of the system can be written as:

\[ M\ddot{q} + B\dot{q} + Kq = CU + D\dot{U} \]  \hspace{1cm} (1)

\( q \in \mathbb{R}^8 \) is the coordinates vector defined by:

\[ q = [z_1, z_2, z_3, z_4, \dot{z}, \phi, \psi]^T \]  \hspace{1cm} (2)

where \((\dot{q}, \ddot{q})\) represent the velocities and accelerations vector respectively. \( U = [u_1, u_2, u_3, u_4]^T \) is the vector of unknown inputs which characterize the road profile, \( M \) is
the inertia matrix, B is related to the damping effects and K is the springs stiffness vector.

We can define a dynamic model of the vehicle as:

$$
\begin{bmatrix}
\dot{v}_x \\
\dot{v}_y \\
\dot{v}_z
\end{bmatrix} = F
$$

(3)

Where \( v = [v_x, v_y, v_z]^T \) is the vehicle velocities vector (along x, y and z axis respectively) and F is the tire/road friction force.

By assuming that the longitudinal forces are proportional to the transversal ones, we expressed these forces as follows:

$$
F_{zf} = \mu F_{zf}
$$

(4)

where \( F_{zf1} \) and \( F_{zf2} \), are the vertical forces of the front and rear wheels respectively. \( F_{xf} \) and \( F_{xf2} \), \( i = 1:2 \) represent the longitudinal forces of the front and rear wheels respectively. \( \mu \) is the adhesion coefficient coming from the consideration of the simplified Burkhart model ([10]):

$$
\mu = C_1(1 - \exp(-C_2 \lambda)) - C_3 \lambda
$$

(5)

where \( C_1, C_2, C_3 \) represent the pneumatic parameters, \( \lambda \) is the longitudinal slip defined by:

$$
\lambda = \frac{v_r - v_s}{\max(v_r, v_s)}
$$

(6)

where \( v_r \) is the wheel velocity.

The wheel angular motion is given by:

$$
\begin{cases}
J_{ei} \ddot{\omega}_i = T_{ei} - rF_{ez}
\\
J_{zi} \ddot{\omega}_i = -rF_{xzi}
\end{cases}
$$

(7)

where \( T_{ei} (i = 1..2) \), is the engine torque. \( r \) is the wheel radius. \( J_{ei} \) and \( J_{zi} \) are the wheel inertia.

**Remark 1:** The engine torque is deduced using vehicle speed and the throttle position, without explicit model of the engine behaviour. The steering and braking angles and the braking are measured.

**ESTIMATION OF THE ROAD PROFILE**

The vertical dynamical model (1) can be written in the state form as follows:

$$
\begin{cases}
\dot{x} = f(x) + CU + D\dot{U} \\
y = h(x)
\end{cases}
$$

(8)

The state vector \( x = (x_{11}, x_{12})^T = (q, \dot{q})^T \).

\( y = q \in \mathbb{R}^k \) is The vector of the measured outputs of the system.

Thus, we obtain:

$$
\begin{cases}
\dot{x}_{11} = x_{12} \\
\dot{x}_{12} = M^{-1}(-Bx_{12} - Kx_{11}) \\
+ M^{-1}(Cx_3 + Dx_4) \\
\dot{x}_3 = x_4 \\
\dot{x}_4 = 0
\end{cases}
$$

(9)

where \( x_3 = U \).

**Remark 2:** Because of low magnitude of accelerations signals (related to the road), we can assume that \( \dot{U} = 0 \).

In the state form, the wheel angular motion becomes:

$$
\begin{bmatrix}
\dot{\zeta} \\
y_1
\end{bmatrix} = f_1(\zeta)
$$

(10)

where \( \zeta = (\xi_1, \xi_2)^T \)

\( y_1 = \left[ w_{i1}, w_{i2}, w_{j1}, w_{j2} \right]^T \)

represents the measured angular velocity vector.

We have:

$$
\dot{\zeta}_1 = \zeta_2 = J^{-1}(\Gamma - R\Psi)
$$

(11)
\[
J = \begin{bmatrix}
J_{r_1} & 0 & 0 & 0 \\
0 & J_{r_2} & 0 & 0 \\
0 & 0 & J_{f_1} & 0 \\
0 & 0 & 0 & J_{f_2}
\end{bmatrix}
\]

\[
\Gamma = [0, 0, T_{r_1}, T_{r_2}]^T, \quad R = rI.
\]

With \( I \in \mathbb{R}^{4 \times 4} \) is the identity matrix and

\[
\Psi = [F_{x_{r_1}}, F_{x_{r_2}}, F_{x_{f_1}}, F_{x_{f_2}}]^T
\]

represents the vector of the longitudinal forces to be estimated.

**OBSERVER DESIGN**

In this paragraph, we develop sliding mode observers in order to estimate the state vector \( x \) and to deduce both the unknown inputs vector \( U \) and its derivative \( \dot{U} \).

We propose the following sliding mode observer:

\[
\begin{align*}
\dot{x}_{1_1} & = \dot{x}_{1_2} + H_1 \text{sign}(\hat{x}_{1_1}) \\
\dot{x}_{1_2} & = -M^{-1}(B \dot{x}_{1_2} + K \hat{x}_{1_1}) \\
\dot{x}_{1_3} & = \dot{x}_{1_4} + H_2 \text{sign}(\hat{x}_{1_2} - \hat{x}_{1_1}) \\
\dot{x}_{1_4} & = H_4 \text{sign}(\hat{x}_{1_1})
\end{align*}
\]

where \( \hat{x}_j \) represents the observed state vector and:

\[
\hat{x}_{1_2} = \hat{x}_{1_2} + H_1 \text{sign}(\hat{x}_{1_1})
\]

\( H_1 \in \mathbb{R}^{8 \times 8} \) and \( H_2 \in \mathbb{R}^{8 \times 8} \) represent positive diagonal gain matrices. \( H_3 \in \mathbb{R}^{4 \times 8} \) and \( H_4 \in \mathbb{R}^{4 \times 8} \) are the gain matrices.

Let us now define an other observer to estimate the vector of longitudinal forces \( \Psi \):

It has the following form:

\[
\begin{align*}
\dot{x}_{1_1} & = J^{-1}(\Gamma - R \dot{\Psi}) + \Lambda_1 \text{sign}(\tilde{\zeta}_1) \\
\dot{\Psi} & = \mu + \Lambda_2 \text{sign}(\tilde{\zeta}_1)
\end{align*}
\]

where \( \Lambda_1 \in \mathbb{R}^{4 \times 4} \) and \( \Lambda_2 \in \mathbb{R}^{4 \times 4} \) represent the positive diagonal gain matrices. \( \mu \) is used to increase the robustness of the observer. It will be defined in the next paragraph.

**CONVERGENCE ANALYSIS**

In this paragraph, we study the convergence of our observers and the stability analysis of the reduced observation errors dynamics.

Let us define the state estimation errors, \( \tilde{x}_i = x_i - \hat{x}_i \) (\( i=1..4 \)), then we can write the dynamics estimation errors as following:

\[
\begin{align*}
\dot{\tilde{x}}_{1_1} & = \tilde{x}_{1_2} - H_1 \text{sign}(\tilde{x}_{1_1}) \\
\dot{\tilde{x}}_{1_2} & = -M^{-1}(B \tilde{x}_{1_2} + K \tilde{x}_{1_1}) \\
\dot{\tilde{x}}_{1_3} & = \tilde{x}_{1_4} - H_2 \text{sign}(\tilde{x}_{1_2} - \tilde{x}_{1_1}) \\
\dot{\tilde{x}}_{1_4} & = H_4 \text{sign}(\tilde{x}_{1_1})
\end{align*}
\]

(12)

In order to study the observer stability and to find the gain matrices \( H_i, \ i=1..4 \), we proceed, step by step, starting to prove the convergence of \( \tilde{x}_{1_1} \) to the sliding surface \( \tilde{x}_{1_1} = 0 \) in finite time \( t_1 \). Then, we deduce some conditions about \( \tilde{x}_{1_2} \) to ensure its convergence towards 0. Finally, we prove that the inputs estimation errors (namely \( \tilde{x}_3 \) and \( \tilde{x}_4 \)) converge to 0.

Let us consider the following Lyapunov function:

\[
V_1 = \frac{1}{2} x_{1_1}^T x_{1_1}
\]

(16)

The time derivative of this function is given by:

\[
\dot{V}_1 = \tilde{x}_{1_1}^T (x_{1_2} - H_1 \text{sign}(\tilde{x}_{1_1}))
\]

(17)

By considering gains matrix \( H_1 = \text{diag}(h_i) \), with \( h_i > |\tilde{x}_{1_2}|, \ i=1..8 \) then \( V_1 < 0 \).

Therefore, from sliding mode theory ([11]), the surface defined by \( \tilde{x}_{1_1} = 0 \) is attractive, leading \( \tilde{x}_{1_1} \) to converge towards \( x_{1_1} \) in finite time \( t_0 \). Moreover, we have \( \tilde{x}_{1_1} = 0 \ \forall \ t \geq t_0 \). Consequently and according to (13), we have the convergence of \( \tilde{x}_{1_2} \) towards \( x_{1_2} \).

According to (15), we have (for \( t \geq t_0 \)):
\[ \text{sign}_{eq}(\hat{x}_i) = H_1^{-1} \hat{x}_2 \] (18)

where \( \text{sign}_{eq} \) represents an equivalent form of the sign function on the sliding surface.

Then, system (15) can be written as follows:

\[
\begin{align*}
\dot{x}_{11} &= \bar{x}_{12} - H_1 \text{sign}_{eq}(\hat{x}_{11}) \\
\dot{x}_{12} &= -M^{-1}(B\bar{x}_{12} + K\hat{x}_{11}) \\
&\quad + M^{-1}(C\bar{x}_3 + D\bar{x}_4) - H_2 H_1^{-1} \hat{x}_{12} \\
\dot{x}_3 &= -H_3 H_1^{-1} \hat{x}_{12} \\
\dot{x}_4 &= -H_4 H_1^{-1} \hat{x}_{12}
\end{align*}
\] (19)

By considering matrix \( H_3 \) components (\( h_{13}, i = 1..4 \)) such that \( h_{13} > |\vec{x}_{14}| \), then system (19) becomes (for \( t \geq t_0 \)):

\[
\begin{align*}
\dot{x}_{11} &= 0 \\
\dot{x}_{12} &= -M^{-1}(B\bar{x}_{12} + K\hat{x}_{11}) \\
&\quad + M^{-1}(C\bar{x}_3 + D\bar{x}_4) - H_2 \text{sign}(\bar{x}_{12}) \\
\dot{x}_3 &= -H_3 \dot{x}_{12} \\
\dot{x}_4 &= -H_4 \dot{x}_{12}
\end{align*}
\] (20)

Now, let us consider a second Lyapunov function \( V_2 \) and its time derivative \( \dot{V}_2 \):

\[
\begin{align*}
V_2 &= \frac{1}{2} \bar{x}_{12}^T M \bar{x}_{12} + \frac{1}{2} \bar{x}_3^T P_1 \bar{x}_3 + \frac{1}{2} \bar{x}_4^T P_2 \bar{x}_4 \\
\dot{V}_2 &= \bar{x}_{12}^T M \dot{\bar{x}}_{12} + \bar{x}_3^T P_1 \dot{x}_3 + \bar{x}_4^T P_2 \dot{x}_4
\end{align*}
\] (21)

where \( P_1 \in \mathbb{R}^{4 \times 4} \) and \( P_2 \in \mathbb{R}^{4 \times 4} \) are the positive diagonal gain matrices, then from (19), \( \dot{V}_2 \) becomes:

\[
\begin{align*}
\dot{V}_2 &= -\bar{x}_{12}^T B \bar{x}_{12} - \bar{x}_{12}^T MH_2 \bar{x}_{12} \\
&\quad + \bar{x}_3^T C \bar{x}_3 + \bar{x}_4^T D \bar{x}_4 - \bar{x}_3^T P H_3 \bar{x}_{12} \\
&\quad - \bar{x}_4^T P_2 H_4 \bar{x}_{12} \\
&\quad - \bar{x}_3^T P H_3 \bar{x}_{12} - \bar{x}_4^T P_2 H_4 \bar{x}_{12}
\end{align*}
\] (22)

By considering that \( H_3 \) and \( H_4 \) are such that:

\[
\begin{align*}
P_1 H_3 H_1^{-1} &= C^T \\
P_2 H_4 H_1^{-1} &= D^T
\end{align*}
\] (23)

We finally obtain:

\[
\dot{V}_2 = -\bar{x}_{12}^T (B + MH_2) \bar{x}_{12}
\] (24)

Recalling that \( B, M \) and \( H_1 \) are positive definite matrices, and by choosing \( H_2 \) of the form:

\[
H_2 = -\bar{x}_{12}^T (B + MH_2) \bar{x}_{12}
\] (25)

such that \( Q = B + MH_2 \) is a positive definite diagonal matrix (with \( Q \in \mathbb{R}^{4 \times 4} \)) then \( \dot{V}_2 < 0 \). Therefore, the surface \( \bar{x}_{12} = 0 \) is attractive, leading \( \bar{x}_{12} \) to converge towards \( x_{12} \). According to (20), we can then deduce that the estimation error of the road profile \( \bar{x}_3 \) and its time derivative \( \bar{x}_4 \) also converge towards 0. The dynamic estimation error of \( \bar{\zeta}_1 \) is given by:

\[
\dot{\bar{\zeta}_1} = -J^{-1} R \bar{\Psi} - \Lambda_1 \text{sign}(\bar{\zeta}_1)
\] (26)

The force estimation error \( \bar{\Psi} \) is defined by:

\[
\dot{\bar{\Psi}} = -\mu - \Lambda_2 \text{sign}(\bar{\zeta}_1)
\] (27)

We consider the following Lyapunov:

\[
\begin{align*}
V_4 &= \frac{1}{2} \bar{\zeta}_1^T P \bar{\zeta}_1 + \frac{1}{2} \bar{\Psi}_1^T P \bar{\Psi}_1
\end{align*}
\] (28)

with \( P \in \mathbb{R}^{4 \times 4} \) is a diagonal positive matrix.

According to (26) and (27), the time derivative of this function gives:

\[
\begin{align*}
\dot{V}_4 &= \bar{\zeta}_1^T \bar{\zeta}_1 + \bar{\Psi}_1^T P \bar{\Psi}_1 \\
&\quad - \bar{\Psi}_1^T \mu - \bar{\Psi}_1^T P \Lambda_2 \text{sign}(\bar{\zeta}_1)
\end{align*}
\] (29)
While choosing the matrix $\Lambda_2$ components $(\Lambda_{12}, i = 1..4)$ such that $\Lambda_{12} < \left[p^{-1} \Psi^{-1} \tilde{z}^T \Lambda_1 \right] \text{ and } 
abla = -\tilde{z}^T J^{-1} R \hat{p}^T$ the equation (29) becomes:

$$\dot{V}_1 = -\tilde{z}_1^T \Lambda_1 \text{sign}(\tilde{z}_1) < 0$$  (30)

Therefore, the surface $\tilde{z}_1 = 0$ is attractive and we have the convergence of $\dot{\zeta}_1$ towards $\zeta_1$.

MAIN RESULTS

In this section, we give some results in order to test and validate our approach. The model parameters are measured. However, the pneumatic parameters, $C_1$, $C_2$, and $C_3$ are not known. To mitigate this disadvantage, we use observers to estimate the longitudinal forces which are related to these parameters. The system outputs are the displacements of the wheels and the chassis, which correspond to signals given by sensors. Different measures are done with the vehicle moving at several speeds. Through figure 2, we present the behaviour of the road profile estimator, for the vehicle moving at average speed of 70km/h. This figure shows the measured and the estimated displacements.

In the first two subplot on top of figure 2, the vertical displacement (z) and the yaw angle (ψ) of the chassis respectively are presented. The bottom of this figure, represent the velocities. We can see that the estimated vertical velocity (\dot{z}) is accurate compared to the true signal coming from sensors. However, some error occurs concerning the estimation of \dot{\psi}. This error is mainly due to sensor calibration. In the figure (1), we notice that the estimated angular velocity of the wheel converges well towards observed one. The figure (3), presents both the measured road profile (coming from LPA instrument) and the estimated one. We can then observe, that the estimated values are quite close to the true ones.
CONCLUSION

In this paper, we have developed sliding mode observers to estimate the states of the system and with the same time the unknown inputs which correspond to the profile of the roadway. The parameters of the system are presumably measured and known. However, the pneumatic coefficients which intervene in the calculation of the longitudinal forces are unknown. This is why, we built an other observer to consider directly these longitudinal forces. One was noticed that the robustness of the sliding mode observers is verified. The profile estimated by our approach is compared to that measured by LPA instrument. The estimation error is near of 2mm.

REFERENCES


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hereby expresses its
sincere gratitude and appreciation to

Hocine Imine

in recognition of a substantial contribution to the

2003 SAE World Congress
March 3-6, 2003

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