Road Profiles Inputs to evaluate loads on the wheels

H. Imine¹², Y. Delanne² and N.K. M'Sirdi¹

¹Laboratoire de Robotique de Versailles, Université de Versailles
10 Avenue de l'Europe, 78140, Vélizy, France

²Laboratoire Central des Ponts et Chaussées
Centre de Nantes, Route de Bouaye, BP 4129-44341 BOUGUENAIS Cedex, France

Email: imine@robot.uvsq.fr

SUMMARY

Vehicle motion simulation accuracy, such as in accident reconstruction or vehicle controllability analysis on real roads, can be obtained only if valid road profile and tire-road friction models are available. Regarding road profiles, a new method based on Sliding Mode Observers has been developed and is compared to LPA measure.

Keywords: Road Profile, APL, Vehicle modelling, loads on the wheels, Sliding Mode Observers.

1. INTRODUCTION

The dynamics of vehicle is directly dependant on tire/road contact forces and torques which are themselves dependant on loads on the wheels and tire/road friction characteristics. To obtain a good evaluation of friction forces and torques in the tire contact patch, it is necessary to have an accurate evaluation of wheels loads. This can be achieved only if relevant road profiles are input to the vehicle dynamic model.

For the purpose of road serviceability, survey and road maintenance, several profilometers have been developed. In a recent European program called FILTER. Some of them have proved to give reliable measurements as compared to the profiles obtained with the reference device [1].

In this paper, we present two methods to evaluate the road profile, namely the longitudinal profile analyser. Method developed by LCPC Laboratories in the 1960's [2] and a recently developed Robotic approach based on Sliding Mode Observers [3], [4].

The objective of this research in that regard was to develop an easily implemented method based on the dynamic response of a vehicle instrumented with cheap sensors so as to give an accurate estimation of the profile along the actual wheel tracks. The method based on sliding mod observers, considers the road profile as unknown inputs of vehicle dynamic system to be estimated. Consequently, the loads on the wheels are evaluated. In this work, some experimental results related to the estimation of the road profile by these methods are shown and discussed to evaluate the robustness of our approach.

A comparison of their relevance to get a good estimate of loads on the wheels, is carried out firstly from the dynamic model developed in the framework of the observers approach and secondly in a validation procedure comparing measured and computed dynamic responses of an instrumented vehicle.

2. THE LONGITUDINAL PROFILE ANALYSER

In this section we present an instrument to measure the road profile, namely the Longitudinal Profile (APL in French). This system includes one or two single-wheel trailers towed at constant speed by a car, and employs a data acquisition system. A ballasted chassis supports an oscillating beam holding a feeler wheel that is kept in permanent contact with the pavement by a suspension and damping system. The chassis is linked to the towing vehicle by a universal-jointed hitch. Vertical movements of the wheel result in angular travel of the beam, measured with respect to the horizontal arm of an inertial pendulum, independently of movements of the towing vehicle (Figure 1).
This measurement is made by an angular displacement transducer associated with the pendulum; the induced electrical signal is amplified and recorded. Rolling surface undulations in a range of plus or minus 100 mm are recorded with wavelengths in ranges from 0.5 to 20 m to 1 to 50 m, depending on the speed of the vehicle [5].

This device has proved to give very precise measurements of profile elevation. Rough measurements have to be processed to get a reliable estimation of the road profile in the measured waveband (phase distortion correction).

3. ESTIMATION OF THE ROAD PROFILE

To implement the sliding mode method, a vehicle model must be assumed ([6], [7]).

3.1. VEHICLE MODELING

The vehicle model is shown in figure 2.

In this part, we are interested in the excitations of pavement and the interaction vehicle/road ([15], [16], [17], [18]). The model is established while making the following simplifying hypotheses:

- The vehicle is rolling with constant speed.
- The wheels are rolling without slip and without contact loss.
3.1.1. VERTICAL MODEL

The vertical motion of the vehicle model can be described by the following equation:

\[
M \ddot{q} + B \dot{q} + K q = \begin{bmatrix} \zeta^T & 0 & 0 & 0 \end{bmatrix}^T
\]

(1) \quad q \in \mathbb{R}^8 \text{ is the coordinates vector defined by:}

\[
q = \begin{bmatrix} z_1, z_2, z_3, z_4, z, \theta, \phi, \psi \end{bmatrix}^T
\]

(2) \quad z_i = 1..4 \text{ is the displacement of the wheel } \dot{z}_i. \text{ The variables } z, \theta, \phi \text{ and } \psi \text{ represent the displacement of the vehicle body, roll angle, pitch angle and the yaw angle respectively.}

\((\dot{q}, \ddot{q})\) represent the velocities and accelerations vectors respectively. \(M \in \mathbb{R}^{8 \times 8}\) is the inertia matrix:

\[
M = \begin{bmatrix} M_i & 0 \\ 0 & M_z \end{bmatrix}, \text{ where } M_i = \text{diag}(m_1, m_2, m_3, m_4) \text{ and } M_z = \text{diag}(J_x, J_y, J_z)
\]

\(m_1, m_2, m_3, m_4\) represent the mass of the wheel \(i\), coupled to the chassis with mass \(m\). \(J_x, J_y, J_z\) are the inertia moments of the vehicle along respectively \(x, y\) and \(z\) axis.

\(B \in \mathbb{R}^{8 \times 8}\) is related to the damping effects:

\[
B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}, \text{ where } B_{11} = \text{diag}(B_{11} + B_{12}, B_{21}, B_{22} + B_{22}, B_{11}, B_{21}, B_{22}) \text{ is a diagonal positive matrix and:}
\]

\[
B_{12} = \begin{bmatrix} -B_1 & C_{16} & C_{17} & 0 \\ -B_2 & C_{26} & C_{27} & 0 \\ -B_3 & C_{36} & C_{37} & 0 \\ -B_4 & C_{46} & C_{47} & 0 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -B_{11} & -B_{12} & B_{13} & -B_{14} \\ -B_{21} & -B_{22} & B_{23} & -B_{24} \\ -B_{31} & -B_{32} & B_{33} & -B_{34} \end{bmatrix}, \quad B_{22} = \begin{bmatrix} C_{55} & C_{56} & C_{57} & 0 \\ C_{65} & C_{66} & C_{67} & 0 \\ C_{75} & C_{76} & C_{77} & 0 \\ C_{85} & C_{86} & C_{87} & C_{88} \end{bmatrix}
\]

The elements of these matrices are defined in appendix.

\(K \in \mathbb{R}^{8 \times 8}\) is the springs stiffness vector:

\[
K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}, \text{ where } K_{11} = \text{diag}(k_1 + k_{11}, k_2 + k_{12}, k_3 + k_{13}, k_4 + k_{14}) \text{ is a diagonal positive matrix and:}
\]

\[
K_{12} = \begin{bmatrix} -k_1 & k_{pr} & k_r & 0 \\ -k_2 & k_{pr} & k_r & 0 \\ -k_3 & k_{pf} & k_r & 0 \\ -k_4 & k_{pf} & k_r & 0 \end{bmatrix}, \quad K_{21} = \begin{bmatrix} -k_1 & -k_2 & -k_3 & -k_4 \\ k_{pr} & -k_{pr} & k_{pf} & -k_{pf} \\ k_r & k_r & -k_r & -k_r \\ k_{pf} & -k_{pf} & k_r & -k_r \end{bmatrix}, \quad K_{22} = \begin{bmatrix} K_{55} & K_{56} & K_{57} & 0 \\ K_{65} & K_{66} & K_{67} & 0 \\ K_{75} & K_{76} & K_{77} & 0 \\ K_{85} & K_{86} & K_{87} & K_{88} \end{bmatrix}
\]

The elements of these matrices are defined in appendix.

In order to estimate the unknown vectors \(U\) and \(\dot{U}\), let us define the variable \(\zeta\) as:

\[
\zeta = C U + D \dot{U}
\]

(3)
where $\zeta \in \mathbb{R}^4$ and $U = [u_1, u_2, u_3, u_4]^T$ is the vector of unknown inputs which characterize the road profile. The system outputs are the displacements of the wheels and the chassis, corresponding to signals measured by the vehicle sensors.

The matrices M, B, K, C and D are defined in appendix.

### 3.1.2. LONGITUDINAL MODEL

The longitudinal force $F_x$ is given by the following equation [8]:

$$F_x = \mu F_z$$

where $\mu$ is the adhesion coefficient and $F_z$ represent the vertical tyre force. The variation in $F_z$ can be calculated using the following formula:

$$F_z = m_i g + K_{z_i}(z_{z_i} - u_i) + B_i(\dot{z}_{z_i} - \dot{u}_i), i = 1..4$$

Where $i$ refer to the position of the wheel.

### 3.2. OBSERVER DESIGN

For our study, we put the model (1) in state equation form while taking as state vector:

$$x_1 = q \text{ and } x_2 = \dot{q} = (x_{21}^T, x_{22}^T)^T \text{ where } x_{21} = [\dot{z}_1, \dot{z}_2, \dot{z}_3, \dot{z}_4]^T \text{ and } x_{22} = [\dot{z}, \dot{\phi}, \dot{\psi}]^T.$$  

Then we obtain:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_{21} &= -M_1^{-1}(B_{11}x_{21} + B_{12}x_{22} + K_{11}x_1 + \zeta) \\
\dot{x}_{22} &= -M_2^{-1}(B_{21}x_{21} + B_{22}x_{22} + K_{22}x_1) \\
y &= [x_{21}^T, x_{22}^T]^T
\end{align*}$$

The matrices $K_{11}$ and $K_{22}$ are defined in $\mathbb{R}^{k \times 8}$.

Before developing the sliding mode observer [9], [10], let us consider the following hypotheses:

1. The state is bounded ($\|x(t)\| < \infty \forall t \geq 0$).
2. The system is the inputs bounded ($\exists$ a constant $\mu \in \mathbb{R}^4$ such as: $\dot{U} < \mu$).
3. The vehicle rolls at constant speed on a defect road of the order of mm, without bumps).

The structure of the proposed observer is triangular [11] having the following form:

$$\begin{align*}
\dot{x}_1 &= \hat{x}_2 + H_1\text{sign}(\hat{x}_1) \\
\dot{x}_{21} &= -M_1^{-1}(B_{11}\hat{x}_{21} + B_{12}\hat{x}_{22} + K_{11}\hat{x}_1 + \hat{\zeta}) + H_2\text{sign}(\bar{x}_{21} - \hat{x}_{21})
\end{align*}$$

where $\hat{x}_1$ represents the observed state vector and $\hat{\zeta}$ is the estimated value of $\zeta$. The variable $\bar{x}_2$ is given in mean average by:

$$\bar{x}_2 = \hat{x}_2 - H_1\text{sign}(\hat{x})$$
\( H_i \in \mathbb{R}^{8 \times 8}, H_{21} \in \mathbb{R}^{4 \times 4} \) and \( H_{22} \in \mathbb{R}^{4 \times 4} \) represent positive diagonal gain matrices.

\( \text{sign}_{eq1} \) is the equivalent of the \text{sign} function in the slide surface \( (\hat{x}_i = x_i - \hat{x}_i = 0) \) [12].

The dynamics estimation errors are given by:

\[
\begin{cases}
\hat{x}_i = \tilde{x}_2 - H_i \text{sign} (\tilde{x}_i) \\
\hat{x}_{21} = -M_i^{-1}(B_{11} \tilde{x}_{21} + K_{11} \tilde{x}_1 + \zeta) - H_{21} \text{sign}_2 (\tilde{x}_{21} - \tilde{x}_{21})
\end{cases}
\]

(9)

### 3.3. CONVERGENCE ANALYSIS

In order to study the observer stability and to find the gains matrices \( H_i, \ i \in \{1,2\} \), we proceed, step by step, starting to prove the convergence of \( \hat{x}_i \) to the sliding surface \( \tilde{x}_i = 0 \) in finite time \( t_i \). Then we deduce some conditions about \( \hat{x}_2 \) to ensure its convergence towards 0.

We consider the following Lyapunov function

\[
V_i = \frac{1}{2} \tilde{x}_i^T \tilde{x}_i
\]

(10)

It can be easily shown that if \( h_{1i} > |\tilde{x}_{i2}|, i \in \{1,2\} \) then \( \dot{V}_i < 0 \). Then the variable \( \tilde{x}_i \) converges towards 0 in finite time \( t_0 \). We obtain in the sliding surface:

\[
\dot{\tilde{x}}_i = \tilde{x}_2 - H_i \text{sign}_{eq1} (\tilde{x}_i) = 0 \Rightarrow \tilde{x}_2 = H_i \text{sign}_{eq1} (\tilde{x}_i)
\]

Then according to (8), we have:

\[
\tilde{x}_2 = x_2
\]

(11)

Then, we obtain \( \tilde{x}_{21} = x_{21} \) (\( x_{22} \) is measured).

The system (9) becomes:

\[
\begin{cases}
\dot{\tilde{x}}_i = 0 \\
\dot{\tilde{x}}_{21} = -M_i^{-1}(B_{11} \tilde{x}_{21} + \zeta) - H_{21} \text{sign}_2 (\tilde{x}_{21})
\end{cases}
\]

(12)

The second step is to study the convergence of \( \hat{x}_2 \).

To study the convergence of \( \hat{x}_{21} \), first consider the following Lyapunov function

\[
V_{21} = \frac{1}{2} \tilde{x}_{21}^T M_i \tilde{x}_{21}
\]

(13)

Deriving this function, we obtain:

\[
\dot{V}_{21} = -\tilde{x}_{21}^T B_{11} \tilde{x}_{21} + \tilde{x}_{21}^T \zeta - \tilde{x}_{21}^T M_i H_2 \text{sign} (\tilde{x}_2)
\]

(14)

Since \( \zeta \) is bounded and during this step, the first condition stays true \( (\dot{\tilde{x}}_i = 0) \) and \( B_{11} \) is a diagonal definite matrix, so we have while choosing the terms of the matrix \( H_2 \) very high, \( \dot{V}_{21} < 0 \). Therefore, the variable \( \tilde{x}_{21} \) converges towards 0 in finite time \( t_i > t_0 \) and then \( \hat{x}_2 = 0 \) [13].

The system (9) becomes \( \forall \ t > t_i \):
The estimation of the unknown vector $\tilde{\zeta}$ is obtained according to (15). We have then:

$$\tilde{\zeta} = \zeta - \hat{\zeta} = M_1^{-1}H_{21}\text{sign}(\hat{x}_{21})$$

(16)

Finally, we get the variable $\zeta$:

$$\zeta = \tilde{\zeta} + M_1^{-1}H_{21}\text{sign}(\hat{x}_{21})$$

(17)

In order to estimate the elements $U_i, i=1..4$ of the unknown vector $U$ and according to (3), we resolve the following equation:

$$\zeta = CU + D \frac{dU}{dt}$$

(18)

When we consider the initial conditions $U(t=0)=0$, we obtain from (15), the unknown input vector $U$ so that:

$$u_i = \frac{\tilde{\zeta}}{c_i} (1 - e^{-\frac{c_i}{d_i}}); \ i=1..4$$

(19)

where $c_i=k_i; i=1..4$ and $d_i=B_i; i=1..4$ are the elements of the matrices $C$ and $D$ given in appendix.

We have discussed in this paper, two methods to estimate the road profile, namely, the LPA measure and the method using sliding mode observers. In the next section, we compare results obtained using these methods.

3.4. ESTIMATION RESULTS

Some tests were carried out at the French Central Laboratory of Roads and Bridges (LCPC) test track with an instrumented car towing two LPA trailers at a constant speed of 72km/h.

The signal measured by a Longitudinal Profile Analyser (LPA) constitutes in this experiment our reference profile.

Figure 3 shows clearly that the estimated displacements of the four wheels converge quickly to the measured ones.
In the first two subplots on top of figure 4, we present respectively the vertical displacement ($z$) and the yaw angle ($\psi$) of the chassis.

Figure 3 – displacements of wheels: estimated and measured

Figure 4 – estimation of displacement of body and yaw angle
The bottom subplots of this figure represent the velocities. We can see that the estimated vertical velocity \( \dot{z} \) is very close to the measured signal. However, the estimation of \( \psi \) is not very good.

A good reconstruction of states enables the estimation of the unknown inputs of the system. Figure 5 presents both the measured road profile (coming from LPA instrument) and the estimated one. We can then observe that the estimated values are quite close to the true ones.

4. EVALUATE LOADS ON THE WHEELS

From the equations (19) and when the unknown inputs \( u_i, i=1..4 \) are estimated, we can deduce the loads on the wheels using the equations (5). The figure (6) represents respectively the friction coefficient and the load on the wheel.
6. CONCLUSIONS

In this paper, we developed a method to estimate the road profile elevation based on sliding mode observers. Compared to the LPA signal, our estimation is correct. It has been shown that by estimating the road profile, we can deduce load on the wheels and know

Regarding our objective in this work, we consider, that if the output vector (vertical acceleration displacement of the wheels and vertical and rotational movement of the vehicle body) is accurate, the sliding mode observers method constitutes a reliable and easily implemented method to estimate the road profile. Consequently, we have a good estimation of the load on the wheels.

REFERENCES

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APPENDIX

\[
C = \begin{bmatrix}
k_{r1} & 0 & 0 & 0 \\
0 & k_{r2} & 0 & 0 \\
0 & 0 & k_{f1} & 0 \\
0 & 0 & 0 & k_{f2}
\end{bmatrix}, \quad D = \begin{bmatrix}
B_{r1} & 0 & 0 & 0 \\
0 & B_{r2} & 0 & 0 \\
0 & 0 & B_{f1} & 0 \\
0 & 0 & 0 & B_{f2}
\end{bmatrix}
\]
The elements of this matrix are given by:

\[
\begin{align*}
K_{55} &= (k_1 + k_2 + k_3 + k_4) \\
K_{66} &= -((k_1 - k_2)p_r + (k_3 - k_4)p_f) \\
K_{77} &= -((k_1 + k_2)r_2 + (k_3 + k_4)r_1) \\
K_{65} &= -((k_1 - k_3)p_r + (k_3 - k_4)p_f) \\
K_{66} &= -(k_1 + k_2 + k_3 + k_4)p_f p_r + (k_{arr} + k_{ar f}) \\
K_{67} &= ((k_1 - k_2)r_2 p_r + (k_3 - k_4)r_1 p_f) \\
K_{75} &= -((k_1 + k_3)r_1 - (k_3 + k_4)r_2) \\
K_{76} &= -((k_1 - k_3)r_2 p_r + (k_3 - k_4)r_1 p_f) \\
K_{77} &= ((k_1 + k_2)r_2^2 + (k_3 + k_4)r_1^2)
\end{align*}
\]

\[
B = \begin{bmatrix}
(B_1 + B_{r1}) & 0 & 0 & 0 & -B_1 & C_{16} & C_{17} & 0 \\
0 & (B_2 + B_{r2}) & 0 & 0 & -B_2 & C_{26} & C_{27} & 0 \\
0 & 0 & (B_3 + B_{f1}) & 0 & -B_3 & C_{36} & C_{37} & 0 \\
0 & 0 & 0 & (B_4 + B_{f2}) & -B_4 & C_{46} & C_{47} & 0 \\
-B_1 & -B_2 & -B_3 & -B_4 & C_{65} & C_{66} & C_{67} & 0 \\
B_1 p_r & -B_2 p_r & B_3 p_f & -B_4 p_f & C_{65} & C_{66} & C_{67} & 0 \\
B_1 r_2 & B_2 r_2 & -B_3 r_1 & -B_4 r_1 & C_{75} & C_{76} & C_{77} & 0 \\
C_{81} & C_{82} & C_{83} & C_{84} & C_{85} & C_{86} & C_{87} & C_{88}
\end{bmatrix}
\]

where:

\[
\begin{align*}
C_{16} &= B_1 p_r \cos(\theta) \\
C_{17} &= B_1 r_2 \cos(\phi) \\
C_{26} &= -B_2 p_r \cos(\theta) \\
C_{27} &= B_2 r_2 \cos(\phi) \\
C_{36} &= B_3 p_f \cos(\theta) \\
C_{37} &= -B_3 r_1 \cos(\phi) \\
C_{46} &= -B_4 p_f \cos(\theta)
\end{align*}
\]
\[ C_{47} = -B_4 r_1 \cos(\phi) \]
\[ C_{55} = (B_1 + B_2 + B_3 + B_4) \]
\[ C_{66} = -((B_1 - B_2)p_r + (B_3 - B_4)p_f) \cos(\theta) \]
\[ C_{67} = -((B_1 + B_2)r_2 + (B_3 + B_4)r_1) \cos(\phi) \]
\[ C_{68} = -((B_1 - B_2)p_r + (B_3 - B_4)p_f) \]
\[ C_{66} = -(B_1 + B_2 + B_3 + B_4)p_f p_r \cos(\theta) \]
\[ C_{67} = -((B_1 - B_2)r_2 p_r - (B_3 - B_4)r_1 p_f) \cos(\phi) \]
\[ C_{78} = -((B_1 + B_2)r_1 p_r - (B_3 + B_4)r_2 p_f) \]
\[ C_{78} = -(B_1 - B_2)r_2 p_r + (B_3 - B_4)r_1 p_f) \cos(\theta) \]
\[ C_{77} = ((B_1 + B_2)r_2^2 + (B_3 + B_4)r_1^2) \cos(\phi) \]
\[ C_{88} = 2(C_{yf}r_1^2 + C_{yr}r_2^2)/\nu \]